SPEED OF SOUND IN TWO-PHASE VAPOR-LIQUID SYSTEMS

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The principal theorems of thermodynamics of irreversible processes are applied to the process of propagation of acoustic waves in a two-phase medium. Expressions are derived which determine the dependence of the sound speed in a vapor-liquid medium on the degree of dryness and the degree of nonequilibrium of different relaxation processes accompanying the propagation of acoustic waves. In the limiting case of equilibrium these expressions reduce to the well-known formulas obtained in equilibrium thermodynamics.

The problem of velocity of propagation of weak perturbations in a two-phase medium in equilibrium had been solved earlier by the methods of equilibrium thermodynamics (for example, [1-3]). The available experimental data are as yet too meager [4-7] and contradictory to make general conclusions; however, the large discrepancies between the results of measurements of sound speed and the computations by the formulas of equilibrium thermodynamics are beyond doubt. A reason for these discrepancies may be the nonequilibrium character of the wave propagation during the measurements of the sound speed. The disagreements among the experimental data obtained by different authors are apparently due to different degrees of equilibrium in the corresponding experiments.

The attempts in [5-8] to derive new theoretical relations can not be considered satisfactory since they are actually based on some prespecified artificial conditions of propagation of acoustic waves, which are not valid in most cases in a real two-phase medium. Thus, in [5-8] a medium is considered in which phase transitions and heat exchange between the phases do not at all occur during the propagation of the sound wave, whereas the momentum exchange between the phases occurs according to the equilibrium condition. Under the same conditions pertaining to the phase transitions and heat exchange in [6, 7] the effect of the nonequilibrium nature of the momentum exchange between the phases on the speed of sound is investigated.

During the propagation of a sound wave in a real vapor-liquid medium, the above processes occur simultaneously in most cases. Hence, for a correct solution of the problem of sound speed in two-phase media a complete analysis of the simultaneous complex effects of all the processes that accompany the propagation of sound wave is necessary. Below, this analysis is done based on the methods of thermodynamics of irreversible processes.

The speed of sound can be regarded as a physical characteristics of a two-phase medium as a whole only when the conditions $d \ll \lambda$, $D \ll \lambda$ are satisfied; here λ is the wavelength, d is the diameter of the particles of the dispersed phase, and D is the distance among the particles. But with respect to the sound wave a two-phase medium behaves as a continuous medium, i.e., the Laplace equation derived from the most general equations for a continuous medium [9] is valid:

$$a^2 = -v^2\left(\frac{\partial p}{\partial v}\right)_s^* \tag{1}$$

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Here the derivative $(\partial p/\partial v)^*_s$ characterizes the relationship between the pressure and the specific volume and a two-phase system in a nonequilibrium adiabatic process.

It should be mentioned that an adiabatic process can be identified with an isentropic one at small deviations from the state of equilibrium [10], including the case of propagation of small perturbations, of which a sound wave is an example.

The determination of the nonequilibrium derivative $(\partial p/\partial v)^*_s$ taking into consideration all nonequilibrium processes occurring simultaneously is possible only by using the methods of thermodynamics of irreversible processes. Therefore, the most complete solution of the problem of speed of sound in two-phase media can be obtained using the thermodynamics of irreversible processes.

A two-phase medium is a thermodynamic system which can be both in equilibrium and nonequilibrium state. The equilibrium state of this system is characterized primarily by its phase, thermal, and mechanical equilibrium.

During the propagation of a sound wave, the equilibrium state of the two-phase medium is disturbed, and the processes occuring in it are, in general, nonequilibrium and of relaxation type.

Each relaxation process can be regarded as a certain internal relaxation [10] characterized by the degree of completeness of the relaxation ξ_j and the affinity R_j. For small departures from the state of equilibrium the phenomenological law of linear dependence of generalized fluxes of the system on the generalized forces holds. In the present case the quantity R_j should be taken as the generalized forces and $d\xi_j/dt$ as the generalized fluxes; here t denotes time.

In the general case the phenomenological law for n mutually independent processes is expressed in the form

$$\frac{d\xi_j}{dt} = \sum_{q=1}^{n} \alpha_{jq} R_q \tag{2}$$

However, it is well known [10] that the matrix of the phenomenological coefficients ξ_j can always be diagonalized by changing the variables and, hence, R_j, i.e., the matrix can be reduced to a form in which $\alpha_{iq} = 0$ for $j \neq q$ and $\alpha_{iq} \neq 0$ for $j = q$. This means that the reactions corresponding to the variables ξ_j chosen in this way are independent and for each reaction Eq. (2) reduces to a simpler form

$$\frac{d\xi_j}{dt} = \alpha_j R_j \tag{3}$$

Expanding R_j in Taylor series around the equilibrium value of the parameter ξ_j°, from (3) we can obtain the equation of relaxation

$$\frac{d\xi_j}{dt} = -\frac{\xi_j - \xi_j^\circ}{\tau_j} \tag{4}$$

where τ_j is the relaxation time of the j-th process.

Hence, each independent process is characterized by a definite relaxation time. For independent processes the concept of relaxation time is obviously devoid of meaning, since in this case (2) is not transformed to form (4). A system in which several processes (dependent or independent) occur simultaneously is characterized by a spectrum of relaxation times τ_j, determined from Eqs. (4) after reducing the parameters ξ_j to independent form [10].

We shall consider the two-phase medium as a system in which n independent relaxation processes occur during perturbation. In contrast to the equilibrium state the state of this system is not determined by two independent thermodynamic parameters. It is necessary to add n further independent parameters ξ_j to these two. Choosing s and v as the independent thermodynamic parameters, we can write

$$p = p(s, v, \xi_1, \xi_2, \ldots \xi_j \ldots \xi_n) \tag{5}$$

For small perturbations the pressure increment is

$$\Delta p = \left(\frac{\partial p}{\partial s}\right)_{v,\xi} \Delta s + \left(\frac{\partial p}{\partial v}\right)_{s,\xi} \Delta v + \sum_{j=1}^{n} \left(\frac{\partial p}{\partial \xi_j}\right)_{s,v} \Delta \xi_j \tag{6}$$

The index ξ in the derivatives indicates that the degree of completeness of all reactions remains constant.

Considering that $\Delta s = 0$ for an adiabatic process, from (6) we obtain

$$\left(\frac{\Delta p}{\Delta v}\right)_s^* = \left(\frac{\partial p}{\partial v}\right)_{s,\xi} + \sum_{j=1}^{n} \left(\frac{\partial p}{\partial \xi_j}\right)_{s,v} \frac{\Delta \xi_j}{\Delta v} \tag{7}$$

The sound wave determines the harmonic character of the variations of the parameters ξ_j; therefore, we have

$$\Delta \xi_j = b_j e^{i\omega t}, \quad \frac{d\xi_j}{dt} = i\omega \Delta \xi_j \tag{8}$$

Comparing (8) and (4), we get

$$\Delta \xi_j = \frac{\Delta \xi_j^{\circ}}{1 + i\omega\tau_j} \tag{9}$$

Substituting (9) into (7) and considering that $(\partial \xi_j^{\circ}/\partial v)_s \equiv (\partial \xi_j/\partial v)_{s,\,R_j}$, we have

$$\left(\frac{\partial p}{\partial v}\right)_s^* = \left(\frac{\partial p}{\partial v}\right)_{s,\xi} + \sum_{j=1}^{n} \frac{1}{i\omega\tau_j + 1} \left(\frac{\partial p}{\partial \xi_j}\right)_{s,v} \left(\frac{\partial \xi_j}{\partial v}\right)_{s,\,R_j} \tag{10}$$

We now make use of the equation for the transformation of derivatives:

$$\left(\frac{\partial p}{\partial v}\right)_{s,\,\xi_{(j-1)},\,R_{(j)}} = \left(\frac{\partial p}{\partial v}\right)_{s,\,\xi_{(j)},\,R_{(j+1)}} + \left(\frac{\partial p}{\partial \xi_j}\right)_{s,v} \left(\frac{\partial \xi_j}{\partial v}\right)_{s,\,R_j} \tag{11}$$

Here the indices $\xi_{(j-1)}$ and $R_{(j)}$ in the derivative denote that the degrees of completeness of all reactions up to and including $j-1$ and the affinities of the remaining reactions starting from j are assumed constant. From (10) and (11) we have

$$\left(\frac{\partial p}{\partial v}\right)_s^* = \left(\frac{\partial p}{\partial v}\right)_{s,\,(n)} + \sum_{j=1}^{n} \frac{1}{i\omega\tau_j + 1} \left[\left(\frac{\partial p}{\partial v}\right)_{s,\,(j-1)} - \left(\frac{\partial p}{\partial v}\right)_{s,\,(j)}\right] \tag{12}$$

The indices ξ and R have been omitted in this expression for the sake of simplicity and the indices in the parentheses containing the derivatives denote the number of reactions for which the degree of completeness is assumed constant. It is understood that the affinities remain constant for the remaining reactions.

Expression (12) gives the complex value of the derivative $(\partial p/\partial v)_s^*$. Hence the solution of the wave equation for the two-phase medium contains a complex wave number. As is well known [9], the imaginary part of the wave number characterizes the absorption of the sound wave, while the real part is related to the speed of its propagation.

Substituting (12) into (1), we obtain the expressions for the complex speed of sound

$$a^2 = a_{(n)}^2 + \sum_{j=1}^{n} \frac{a_{(j-1)}^2 - a_{(j)}^2}{i\omega\tau_j + 1} \tag{13}$$

For small absorption per wavelength the real part of (13) represents the phase velocity of the wave

$$a^2 = a_{(n)}^2 + \sum_{j=1}^{n} \frac{a_{(j-1)}^2 - a_{(j)}^2}{1 + \omega^2\tau_j^2} \tag{14}$$

Let us elucidate the meaning of the quantities $a_{(n)}$, $a_{(j)}$, and $a_{(j-1)}$. In Eq. (6) the derivatives obviously refer to the unperturbed state, when $\xi = 0$, $R = 0$. Hence the condition $\xi_j = \text{const}$ is equivalent to $\xi_j = 0$, i.e., in general the corresponding reaction does not proceed – it is slowed down. Similarly $R_j = \text{const}$ is equivalent to $R_j = 0$, i.e., the corresponding reaction proceeds as in equilibrium. Thus, $a_{(n)}$ is the speed of sound in the two-phase medium, where all the relaxation processes are retarded; $a_{(j)}$ is the speed of sound; the processes up to and including j are retarded; and the remaining processes occur as in equilibrium. The quantity $a_{(j-1)}$ has a similar meaning, but $(j-1)$ processes are assumed to be retarded.

In principle, relation (14) takes account of all the processes occurring simultaneously in a two-phase medium. It is seen from (14) that the speed of sound depends significantly on the quantity $\omega\tau_j$. In particular, if $\omega\tau_j \gg 1$ for all relaxation processes (very high frequency), then (14) gives $a = a_{(n)}$, i.e., the upper boundary of the dispersion. On the other hand, if $\omega\tau_j \ll 1$ for all processes (which is possible for sufficiently low frequencies), then we have $a = a_0$, where a_0 is the speed of sound for the equilibrium character of all the processes (the lower boundary of dispersion). But it is exactly for these conditions that the formula for the speed of sound in two-phase media is derived by the method of equilibrium thermodynamics.

Thus, at the lower boundary of dispersion (14) goes over into the usual equilibrium thermodynamic formula. It must be mentioned that in [6, 7] this limiting transition is not obtained, as a result of which the authors make an unjustified conclusion that the equilibrium formulas derived in [1, 2] are inapplicable even at the lower boundary of dispersion. Actually, in a two-phase system, as in any other medium, an equilibrium propagation of sound waves is possible theoretically, and then the equilibrium formulas are obviously valid. It is well known that in the case of single-phase media the formulas of equilibrium thermodynamics relating the speed of sound with different thermodynamic parameters are also valid only at the lower boundary of dispersion. However, in single-phase media dispersion is a consequence of relaxation processes occurring at molecular level and characterized by relatively small relaxation times. Therefore, in practice the lower boundary of dispersion is realized in a sufficiently wide range of frequencies, which makes it possible to obtain the so called "thermodynamic" value of the sound speed (it would be more correct to use the term "equilibrium") in the experimental investigations relatively easily.

In two-phase media macroscopic relaxation processes, usually characterized by very large relaxation times, have a significant effect on dispersion. Therefore, for two-phase media the lower boundary of dispersion is realized in most cases in a relatively narrow range of small frequencies. Thus, even for two-phase media the concept of the thermodynamic speed of sound has completely real significance, if the conditions $\omega\tau_j \ll 1$ are satisfied. In practice the fulfilment of these conditions can be ensured by decreasing the size of the particles of the dispersed phase, since in doing so τ_j are obviously reduced. The irreducibility to equilibrium formulas in [6, 7] is accounted for by the fact that the authors prespecify the nonequilibrium conditions, assuming the processes of phase transitions and heat exchange to be retarded and considering the dispersion of the speed of sound depending only on a single value $\omega\tau_g$; τ_g is the relaxation time of the momentum exchange between the phases.

It is evident from (14) that such a formulation of the problem will be correct only in the case where τ_g differs from the relaxation times of all the remaining processes by a few orders of magnitude.

In (14) the quantities $a_{(j)}$ are functions of the degree of dryness and pressure. The corresponding expressions can be obtained from an investigation of the properties of a conditional two-phase medium, in which a definite number of independent relaxation processes are retarded.

The processes of phase change and heat exchange in a vapor-liquid medium are inseparably linked with each other; therefore, it is necessary to consider two other processes equivalent to them but mutually independent.

It is well known that the vapor formation and condensation in the volume of a vapor-liquid medium occurs only on the surface of the dispersed phase particles. Hence, the surface temperature is always equal to the saturation temperature T_s, while the rate of phase change depends on the relation between the rates of heat exchange (i.e., the temperature equalization) inside the particle and in the main phase surrounding it.

It is obvious that at constant surface temperature of the particle the processes of temperature equalization in the particle and the main phase occur independently. At the same time these two processes almost completely determine both the phase transition and the heat exchange between the phases. During the propagation of small-amplitude waves, these processes occur independently of the process of equalization of the rates of phase migration.

Let us derive expressions for $a_{(1)}$, $a_{(2)}$, $a_{(3)}$ in accordance with known relaxation processes:

1) temperature equalization to T_s in the main phase;

2) temperature equalization to T_s in the particle;

3) equalization of the rates of phase migration.

We shall consider a "vapor–drops" system. From the equation of additivity of the specific volume and entropy

$$v = (1 - x)\, v' + xv'', \quad s = (1 - x)\, s' + xs'' \tag{15}$$

where x is the degree of dryness, v', s' and v'', s'' are specific volume and entropy of the liquid phase and vapor, we obtain

$$\left(\frac{\partial v}{\partial p''}\right)_s^* = (1 - x)\left(\frac{\partial v'}{\partial p'}\right)_s^* \frac{dp'}{dp''} + x\left(\frac{\partial v''}{\partial p''}\right)_s^* + (v'' - v')\left(\frac{\partial x}{\partial p''}\right)_s^* \tag{16}$$

$$ds = (1 - x)\, ds' + x ds'' + (s'' - s')\, dx \tag{17}$$

In the general case the pressures of the phases are not identical due to the surface forces. In Eq. (16) the pressure of the main phase, i.e., p'', is taken as the reference pressure. The derivatives denoted by an asterisk characterize nonequilibrium processes.

We make use of the relations among the derivatives valid for each phase

$$\left(\frac{\partial v'}{\partial p'}\right)_s^* = \left(\frac{\partial v}{\partial p}\right)_s + \left(\frac{\partial v}{\partial s}\right)_p' \left(\frac{\partial s'}{\partial p'}\right)_s^*, \quad \left(\frac{\partial v''}{\partial p''}\right)_s^* = \left(\frac{\partial v}{\partial p}\right)_s'' + \left(\frac{\partial v}{\partial s}\right)_p'' \left(\frac{\partial s''}{\partial p''}\right)_s^* \tag{18}$$

where the equilibrium derivatives $(\partial v/\partial p)_s$ and $(\partial v/\partial s)_p$ are taken from the side of the single-phase region and also Maxwell's equation

$$\left(\frac{\partial v}{\partial s}\right)_p = \left(\frac{\partial T}{\partial p}\right)_s \tag{19}$$

and transform (16) to the following form:

$$\begin{aligned}\left(\frac{\partial v}{\partial p''}\right)_s^* = (1 - x)\left(\frac{\partial v}{\partial p}\right)_s' \frac{dp'}{dp''} + x\left(\frac{\partial v}{\partial p}\right)_s'' + (1 - x)\left(\frac{\partial T}{\partial p}\right)_s' \left(\frac{\partial s'}{\partial p''}\right)_s^* \\ + x\left(\frac{\partial T}{\partial p}\right)_s'' \left(\frac{\partial s''}{\partial p''}\right)_s^* + (v'' - v')\left(\frac{\partial x}{\partial p''}\right)_s^*\end{aligned} \tag{20}$$

Furthermore, it follows from (17) that

$$\left(\frac{\partial x}{\partial p''}\right)_s^* = -\frac{T}{r}\left[(1 - x)\left(\frac{\partial s'}{\partial p''}\right)_s^* + x\left(\frac{\partial s''}{\partial p''}\right)_s^*\right] \tag{21}$$

where r is the latent heat of vaporization.

Thus, the problem of determining the nonequilibrium derivative $(\partial v/\partial p)_s^*$ reduces to the determination of the nonequilibrium derivatives $(\partial s'/\partial p'')_s^*$ and $(\partial s''/\partial p'')_s^*$ under specified conditions. If only the first relaxation process, i.e., the heat exchange in the vapor surrounding the drop, is retarded and the temperature equalization within the drop occurs as in equilibrium for a given p'', then the temperature of the entire drop is always equal to T_s. For independent variables p, s the elementary temperature increment of the drop is

$$dT' = dT_s = \left(\frac{\partial T}{\partial p}\right)_s' dp' + \left(\frac{\partial T}{\partial s}\right)_p' ds' = \left(\frac{\partial T}{\partial p}\right)_s' dp' + \frac{T}{c_p'} ds' \tag{22}$$

Hence, we have

$$\left(\frac{\partial s'}{\partial p''}\right)_{(1)} = \frac{c_p'}{T}\left[\frac{dT_s}{dp''} - \left(\frac{\partial T}{\partial p}\right)_s' \left(\frac{dp'}{dp''}\right)_{(1)}\right] \tag{23}$$

The derivative dT_s/dp'' is determined by the generalized Classius–Clapeyron equation.

Making use of the well known relations

$$\begin{gathered}\left(\frac{\partial T}{\partial p}\right)_s = \frac{T}{c_p}\left(\frac{\partial v}{\partial T}\right)_p, \quad \left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial v}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_v \\ c_p - c_v = T\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial v}{\partial T}\right)_p\end{gathered} \tag{24}$$

the derivative $(\partial T/\partial p)_s$ can be transformed to the form

$$\left(\frac{\partial T}{\partial p}\right)_s = \frac{v}{a}\left[\frac{T}{c_p}(k-1)\right]^{1/2} \tag{25}$$

Considering that by stipulation there is no heat transfer in the vapor, i.e., $ds'' = 0$, from (1), (20), (21), (23), and (25) we obtain

$$a_{(1)}^2 = v^2\left\{x\,\frac{v''^2}{a''^2} + (1-x)\left[k'\,\frac{v'^2}{a'^2}\left(\frac{dp'}{dp''}\right)_{(1)} + c_p{}'\,\frac{v''-v'}{r}\,\frac{dT_s}{dp''} - \frac{v'}{a'}\left(c_p{}'\,\frac{k'-1}{T}\right)^{1/2}\left[\frac{dT_s}{dp''} + T\,\frac{v''-v'}{r}\left(\frac{dp'}{dp''}\right)_{(1)}\right]\right]\right\}^{-1} \tag{26}$$

The quantity $(dp'/dp'')_{(1)}$ depends on the drop sizes and can be determined from the Laplace equation

$$p' = p'' + \frac{4\sigma}{d} \tag{27}$$

where σ is the surface tension. Differentiation of (27) gives

$$\frac{dp'}{dp''} = 1 + \frac{4}{d}\,\frac{d\sigma}{dp''} - \frac{4\sigma}{3v'd}\left(\frac{\partial v'}{\partial p'}\right)_s^{*}\frac{dp'}{dp''} \tag{28}$$

After some transformation with the use of the expressions presented above we obtain

$$\left(\frac{dp'}{dp''}\right)_{(1)} = \left\{1 + \frac{4}{d}\left[\frac{d\sigma}{dT} - \frac{\sigma}{3a'}\left(c_p{}'\,\frac{k'-1}{T}\right)^{1/2}\right]\frac{dT_s}{dp''}\right\}\left(1 - \frac{4\sigma v'k'}{3a'^2 d}\right)^{-1} \tag{29}$$

If both primary processes, i.e., the heat exchange in the drop and the vapor, are retarded, then it is obvious that $ds' = ds'' = dx = 0$; and from (20) and (1) we obtain

$$a_{(2)}^2 = v^2\left[x\,\frac{v''^2}{a''^2} + (1-x)\,\frac{v'^2}{a'^2}\left(\frac{dp'}{dp''}\right)_{(2)}\right]^{-1} \tag{30}$$

From (28) and (18) we have

$$\left(\frac{dp'}{dp''}\right)_{(2)} = \left(1 + \frac{4}{d}\,\frac{d\sigma}{dT}\,\frac{dT_s}{dp''}\right)\left(1 - \frac{4\sigma v'}{3a'^2 d}\right)^{-1} \tag{31}$$

To determine $a_{(3)}$, we consider an elementary volume of humid vapor δV containing a single drop and characterized by the same mass concentration of the phases as in the system as a whole. In the general case the rates of phase migration are different, and frictional forces appear between the drops and the vapor surrounding it. At small Reynolds numbers (and this condition is satisfied in the case of small perturbations) the frictional force is proportional to the difference of the rates in accordance with the Stokes law [11]. We write the equations of motion of the elementary volume $\delta V''$ of the vapor and the drop whose volume is $\delta V'$ as follows:

$$\begin{aligned}
\frac{1}{v''}\,\frac{\partial^2 u''}{\partial t^2}\,\delta V'' &= -\frac{\partial p''}{\partial y}\,\delta V'' - \beta\left(\frac{\partial u''}{\partial t} - \frac{\partial u'}{\partial t}\right)\\
\frac{1}{v'}\,\frac{\partial^2 u'}{\partial t^2}\,\delta V' &= -\frac{\partial p''}{\partial y}\,\delta V' + \beta\left(\frac{\partial u''}{\partial t} - \frac{\partial u'}{\partial t}\right)
\end{aligned} \tag{32}$$

Here u' and u'' are the displacements of the drop and the vapor, respectively; β is the coefficient of proportionality for the Stokes force; and y is the direction of propagation of the plane wave.

Considering that for harmonic oscillations

$$\frac{\partial u}{\partial t} = i\omega u \tag{33}$$

and the deformation of each phase is adiabatic by assumption, from Eqs. (32) we obtain

$$-\omega^2 u'' = v\,\frac{\beta v v' + i\omega x v''\delta V'}{\beta v' + i\omega x\delta V'}\left[x\left(\frac{\partial v}{\partial p}\right)_s'' + (1-x)\left(\frac{\partial v}{\partial p}\right)_s'\left(\frac{dp'}{dp''}\right)_{(2)}\right]^{-1}\frac{\partial^2 u''}{\partial y^2} \tag{34}$$

Equation (34) has the form of the usual wave equation for a harmonic wave [9]. Hence the coefficient of the term $\partial^2 u''/\partial y^2$ represents the square of the complex sound speed which would characterize a two-phase medium with retarded processes of temperature equalization in the presence of relaxation of momentum exchange. As expected, for $\omega \to 0$ the expression for the complex sound speed goes over into Eq. (30) for $a^2_{(2)}$. For $\omega \to \infty$ we obtain the formula for $a_{(3)}{}^2$

$$a^2_{(3)} = vv''\left[x\,\frac{v''^2}{a''^2} + (1-x)\frac{v'^2}{a'^2}\left(\frac{dp'}{dp''}\right)_{(2)}\right]^{-1} \tag{35}$$

If we disregard the effect of the surface forces and assume that each phase is incompressible and the vapor is ideal, then (35) goes over into the expression derived in [6] under these assumptions and the condition of absence of equalization of the rates of phase migration.

The expressions for a "liquid–vapor bubbles" system can be obtained in a similar way:

$$a^2_{(1)} = v^2\left\{(1-x)\frac{v'^2}{a'^2} + x\left[k''\frac{v''^2}{a''^2}\left(\frac{dp''}{dp'}\right)_{(1)} + c_p{}''\frac{v''-v'}{r}\frac{dT_s}{dp'} - \frac{v''}{a''}\left(c_p{}''\frac{k''-1}{T}\right)^{1/2}\left[\frac{dT_s}{dp'} + T\,\frac{v''-v'}{r}\left(\frac{dp''}{dp'}\right)_{(1)}\right]\right]\right\}^{-1} \tag{36}$$

$$a^2_{(2)} = v^2\left[x\,\frac{v''^2}{a''^2}\left(\frac{dp''}{dp'}\right)_{(2)} + (1-x)\,\frac{v'^2}{a'^2}\right]^{-1} \tag{37}$$

$$a^2_{(3)} = vv'\left[x\,\frac{v''^2}{a''^2}\left(\frac{dp''}{dp'}\right)_{(2)} + (1-x)\frac{v'^2}{a'^2}\right]^{-1} \tag{38}$$

In the case of negligible effect of the surface forces, (30) and (37) coincide with the formula derived in [5] under the condition of absence of mass and heat exchange between the phases. If it also assumed that the liquid phase is incompressible and the vapor is ideal, then (30) and (37) go over into the formula derived in [8] under the same assumptions.

It is clear from the preceding discussion that $a_{(2)}$ is a purely arbitrary quantity necessary for calculating the real speed of sound from Eq. (14). It is not surprising, therefore, that the values of sound speed computed from the formulas derived in [5] and [8] differ appreciably from the experimental values by the authors' own admission. It is evident from Eq. (14) that the equality $a = a_{(2)}$ can occur only when the conditions $\omega\tau_1 \gg 1$, $\omega\tau_2 \gg 1$, $\omega\tau_3 \ll 1$ are fulfilled.

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